- ***Candidates should be able to :***

  - ***Define*** *capacitance and the farad.*
  - ***Select*** *and* ***use*** *the equation :* $Q = CV$
  - ***State*** *and* ***use*** *the equation for the total capacitance of two or more capacitors* ***in series.***
  - ***State*** *and* ***use*** *the equation for the total capacitance of two or more capacitors* ***in parallel.***
  - *Solve circuit problems with capacitors involving* ***series*** *and* ***parallel*** *circuits.*
  - *Explain that the area under a potential difference against charge graph is equal to the energy stored by a capacitor.*
  - *Select and use the equations for a charged capacitor :*

$$W = \tfrac{1}{2}QV \qquad W = \tfrac{1}{2}CV^2$$

  - *Sketch graphs that show the* ***variation with time*** *of* ***potential difference, charge*** *and* ***current*** *for a capacitor discharging through a resistor.*
  - ***Define*** *the* ***time constant*** *of a circuit.*
  - *Select and use :* $\text{Time constant} = CR$

  - *Analyse the* ***discharge of a capacitor*** *using equations of the form :*

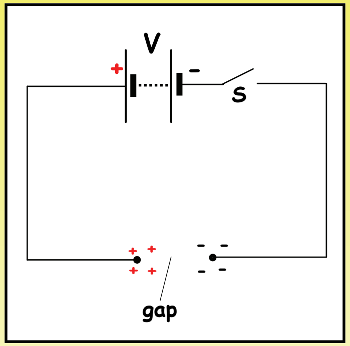
$$x = x_0 e^{-t/CR}$$

  - *Explain* ***exponential decays*** *as having a* ***constant-ratio*** *property.*
  - ***Describe*** *the uses of capacitors for the storage of energy in applications such as* ***flash photography, lasers used in nuclear fusion and as back-up power supplies for computers.***

INTRODUCTION

- *Electric current is a flow of charge, usually carried by electrons. In a circuit the electrons drift along the conductors which make up the circuit. If there is a gap in the circuit, we say that there is no current, but this is not completely true.*

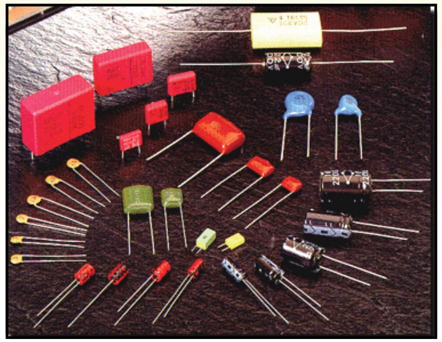
In the circuit shown opposite, when ***S*** *is closed electrons are caused to flow briefly in a* ***clockwise*** *direction.* ***Negative*** *charge accumulates on one side of the gap which repels* ***electrons*** *on the other side. So* ***negative*** *charge builds up on one side and* ***positive*** *charge on the other. The flow quickly stops when the p.d. across the gap (due to the separated charges) becomes equal to the supply voltage,* ***V.***



We can think of the wires on either side of the gap as ***'storing'*** *a tiny amount of charge. This charge storage is the basis of a device called a* ***CAPACITOR.***

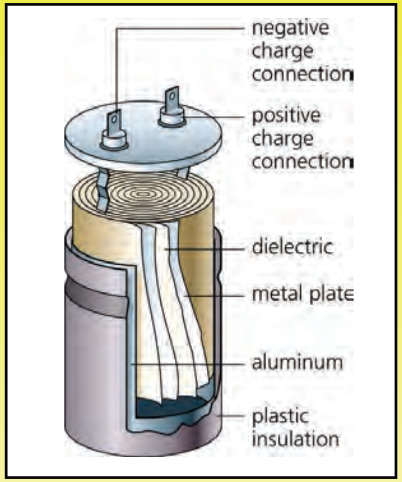
A CAPACITOR is an arrangement of conductors and insulators designed to store electrical charge.

Although capacitors come in a huge variety of different types, shapes and sizes, they all basically consist of two metal plates separated by an insulating material which is called the ***dielectric.***



- *If capacitors were made in their basic form of two flat conducting plates separated by an insulator, they would be excessively large and cumbersome.*

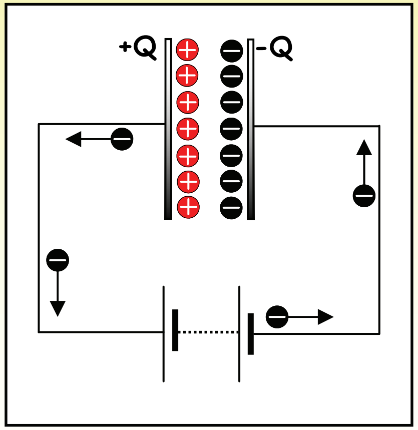
  In order to make capacitors of a size suitable for connection in a circuit, the metal plates and the dielectric are rolled into a cylindrical shape as shown in the diagram opposite.



CHARGING OF A PARALLEL-PLATE CAPACITOR

- *Two parallel, metal plates placed close to each other form a capacitor. When such a capacitor is connected to a battery, one of the plates **gains electrons** and so becomes **negatively charged.***

  *This causes an equal number of electrons to be **repelled** from the other plate, which then becomes **positively** charged. The arrival of electrons at one plate and the repulsion of electrons from the other occurs simultaneously.*



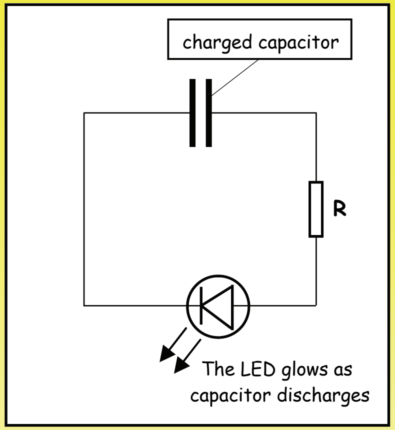
*If one plate stores charge –**Q**, the other stores charge +**Q** and we say that charge **Q** is stored.*

*The amount of charge stored depends on the **POTENTIAL DIFFERENCE (VOLTAGE)** of the supply to which the plates are connected.*

- *Once a capacitor has been **charged**, it can be **discharged** by disconnecting it from the supply and connecting the leads together.*

  We can observe a discharge by connecting a charged capacitor to an LED through a protective resistor.

  The LED glows as the capacitor discharges.



CAPACITANCE

- *The term **CAPACITANCE** is used in order to quantify the amount of charge which can be stored by a given capacitor.*

The **CAPACITANCE** (***C***) of a capacitor is the amount of **charge stored per unit potential difference** across it.

$$\text{CAPACITANCE} = \frac{\text{CHARGE}}{\text{POTENTIAL DIFFERENCE}}$$

$$C = \frac{Q}{V}$$

C (F), Q (C), V (V)

POINTS TO NOTE

- *The **CHARGE (Q)** stored in a capacitor is directly proportional to the **CAPACITANCE (C)** of the capacitor and the **POTENTIAL DIFFERENCE (VOLTAGE) (V)** across it.*

$$Q = CV$$

- *The unit of **capacitance** is the* **FARAD (F)**

1 farad = 1 coulomb per volt
$$1\ F = 1\ C\ V^{-1}$$

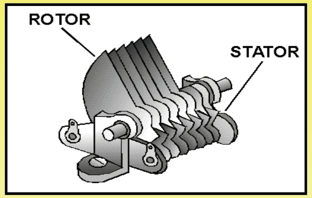
- ***1 farad** is a very large capacitance value and practical capacitors have much smaller capacitances. For this reason smaller sub-multiples are used.*

1 millifarad (mF)	=	10^{-3} F
1 microfarad (μF)	=	10^{-6} F
1 nanofarad (nF)	=	10^{-9} F
1 picofarad (pF)	=	10^{-12} F

USES OF CAPACITORS

- *Electrolytic capacitors (consisting of an electrolyte-soaked paper sandwiched between two aluminium strips) are used in power supply circuits so as to even out voltage fluctuations.*

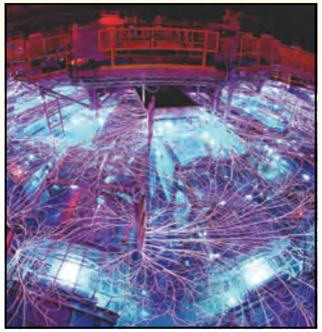
- *Variable air capacitors (interleaving metal plates having an air gap between them) are used in radios so as to tune in to different stations. As the tuning dial is turned, the area of overlap between the plates changes, varying the capacitance and so enabling each station to be tuned in.*



- *Small capacitors are sometimes used to power a **camera flash**. When the capacitor discharges rapidly, the energy stored in it is converted into **heat and light** in the bulb. The charged capacitor releases around **0.5 joule** of energy in about **0.5 millisecond**, giving a power output of about **1 kilowatt!***

- *When there is a brief loss of power to a computer, a capacitor within the computer can supply temporary back-up power so as to prevent data loss.*

- *In the **Z machine** in New Mexico, a laser capable of unleashing an astonishing 10^{15} **W** of power is being used to cause the fusion of nuclei. This astonishing power output is produced by a **60 MJ bank of high-voltage capacitors** which are slowly charged and then discharged in a **few billionths** of a second. Although it is still at the research stage, the Z machine may well be the key which unlocks the door to sustained power from nuclear fusion.*



- PRACTICE QUESTIONS (1)

1 (a) How much **charge** is stored in a **250 μF** capacitor charged up to **15 V**? Give your answer in **μC** and in **C**.

(b) What is the **capacitance** of a capacitor that stores **12 mC** of charge when it is charged to **600 V**? Give your answer in **F**, **μF** and **pF**.

(c) Calculate the **average current** needed to charge a 50 **μF** to **100 V** in a time of 1.5×10^{-2} **s**.

2 A **40 μF** capacitor is charged to a potential difference of **24 V** using a constant current of **4.0 μA**. Calculate :

(a) The **total charge** stored on the capacitor at **24 V**.

(b) The **time taken** to charge the capacitor.

3 A capacitor is charged using a constant current of **24 μA** to a p.d. of **4.2 V** in a time of **38 s**. The same capacitor is then charged from **4.2 V** by means of a constant current of **14 μA** for a further time of **50 s**. Calculate :

(a) The **charge stored** at a p.d. of **4.2 V**.

(b) The **capacitance** of the capacitor.

(c) The **extra charge stored** by the **14 μA** current.

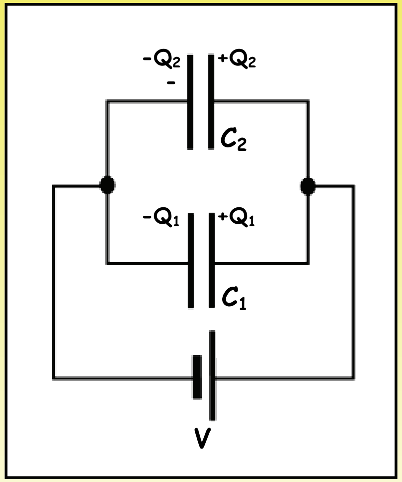
(d) The **p.d. after the extra charge was stored.**

COMBINATION OF CAPACITORS

- Capacitor combinations are used in many electrical circuits and like other components, they may be connected in **parallel** or in **series**.

1. CAPACITORS IN PARALLEL

- Consider two capacitors C_1 and C_2 connected in **parallel** and having a supply of potential difference **V** connected to them as shown in the diagram opposite.
  Components connected in parallel have the **same p.d.**, so the **charge** stored by each capacitor is :



$Q_1 = C_1V$ and $Q_2 = C_2V$

So **total charge stored (Q)** is :

$Q = Q_1 + Q_2 = C_1V + C_2V$

So the **total capacitance (C)** is :

$$C = \frac{Q}{V} = \frac{C_1V + C_2V}{V} = C_1 + C_2$$

For any number **(n)** of capacitors connected in **parallel**, the **total capacitance (C)** is given by :

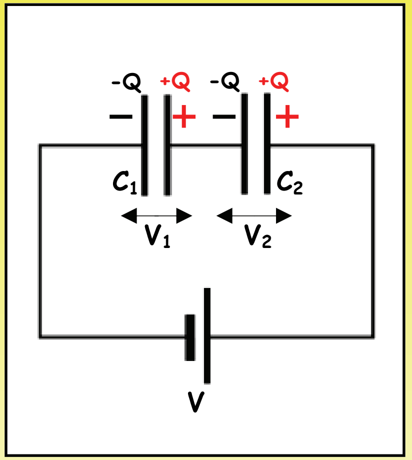
$$C = C_1 + C_2 + \ldots\ C_n$$

- For two capacitors in parallel, the ratio of the charge stored by each capacitor is : $Q_1/Q_2 = C_1V/C_2V = C_1/C_2$

So, the ratio of the charge stored by the capacitors = the capacitance ratio C_1/C_2

2. CAPACITORS IN SERIES

- Consider two capacitors C_1 and C_2 connected in **series** and having a supply of potential difference **V** connected to them as shown in the diagram opposite.



Components in series store the **same charge**, so the charge is the same on each capacitor **(=Q)***.

The **p.d.** across each capacitor is :

$V_1 = \frac{Q}{C_1}$ and $V_2 = \frac{Q}{C_2}$

The **total p.d. (V)** across the capacitors is :

$V = V_1 + V_2 = Q/C_1 + Q/C_2$

The **total capacitance,** $C = Q/V$ so $V = Q/C$

Therefore : $Q/C = Q/C_1 + Q/C_2$

Dividing throughout by Q : $1/C = 1/C_1 + 1/C_2$

For any number **(n)** of capacitors connected in **series,** the **total capacitance (C)** is given by :

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots\ldots \frac{1}{C_n}$$

- For two capacitors in **series,** the ratio of the p.ds ,

$V_1/V_2 = Q/C_1 \div Q/C_2 = C_2/C_1$

So, the ratio of the p.ds across the two capacitors = the inverse ratio of the capacitances C_2/C_1

- **PRACTICE QUESTIONS (2)**

1 (a) Calculate the **total capacitance** of two **100 μF** capacitors connected in **parallel**.

(b) Calculate the **charge** they store when charged to a p.d. of **20 V**.

2 Consider two capacitors, C_1 = **200 μF** and C_2 = **500 μF** connected in **parallel** across a **10 V** supply.

Calculate the **charge stored by each capacitor** and then work out the **total charge stored.** Then show that the **total capacitance** of the two capacitors in parallel is **700 μF.**

3 A capacitor of value **50 μF** is required, but the only values available to you are **10 μF**, **20 μF** and **100 μF**. Give **two** possible ways in which you could get the required value.
You may use more than one of each of the available capacitance values.

4 Calculate the **combined (total) capacitance** of a **300 μF** capacitor and a **600 μF** capacitor connected in **SERIES.**

5 Calculate the **combined (total) capacitance** of three capacitors having capacitances of **200 μF, 300 μF and 600 μF**, connected in **SERIES.**

6 A **3.0 V** battery is connected to a **2.0 μF** capacitor **in parallel** with a **3.0 μF** capacitor.
Sketch the circuit diagram and calculate :

(a) The **combined capacitance** of the two capacitors,

(b) The **charge stored** and the **p.d. across each capacitor**.

7 A **4.0 μF** capacitor **in series** with a **10.0 μF** capacitor are connected to a **6.0 V** battery. A **2.0 μF** capacitor is then connected to the battery **in parallel** with the two capacitors in series.

(a) Sketch the circuit diagram for this arrangement and calculate its **total capacitance**.

(b) Calculate the **charge** and **p.d.** for each capacitor in the arrangement.

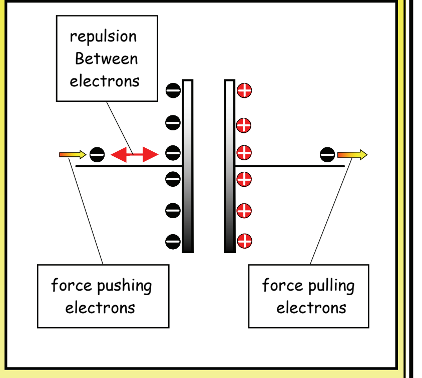
8 You are given three capacitors having capacitances of **10 μF**, **20 μF** and **40 μF**.

Sketch the **six** different combinations which are possible **using all three** capacitors and calculate the **total capacitance** of each combination.

ENERGY STORED IN A CHARGED CAPACITOR

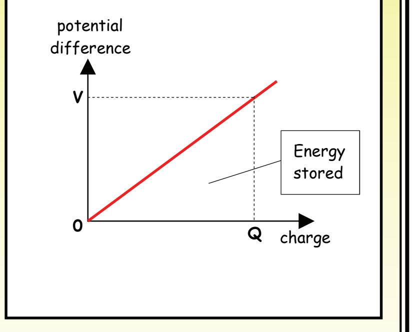
- *When a capacitor is charged* ***work is done*** *to push electrons onto one plate and off the other plate.*

  At first there is only a small amount of negative charge on the left hand plate and so the force repelling additional electrons is small. As the charge stored increases, the repulsion force increases and so a greater amount of work has to be done to increase the quantity of charge stored.



- *Since* ***V = Q/C, V is proportional to Q****, so a graph of V against Q is a straight line.*

  Consider a capacitor of capacitance ***(C)*** *which gets a charge* ***(Q)*** *when the p.d. across it is* ***(V)****.*



Energy stored in the capacitor = Work done in charging

= **Area enclosed by the V/Q graph**

= $\frac{1}{2}$ **Q V**

The gradient of the V/Q graph = 1/CAPACITANCE (C)

If the p.d. across a capacitor of **capacitance (C)** changes by an amount **(V)** when a quantity of **charge (Q)** flows into (or out of) it, the **energy stored** (or given out) by the capacitor **(W)** is given by :

$$W = \frac{1}{2} QV$$

(J) (C) (V)

And since $Q = CV$, $W = \frac{1}{2} CV \times V = \frac{1}{2} CV^2$

$$W = \frac{1}{2} CV^2$$

(J) (F) (V)

And since $V = Q/C$, $W = \frac{1}{2} Q \times Q/C = \frac{1}{2} Q^2/C$

$$W = \frac{1}{2} Q^2/C$$

(J) (C) (F)

- The **energy of a charged capacitor** is stored in the **electric field between the plates**.
- If the capacitor is charged from a battery (or similar source), the charge **(Q)** flows at **constant p.d. (V)**. In this case, the energy drawn from the battery is equal to **QV** (i.e. twice the energy stored in the capacitor). The 'missing' energy is dissipated as heat in the connecting wires.

• PRACTICE QUESTIONS (3)

1 Calculate the **charge** and **energy stored** in each of the following cases :

(a) A **5000 μF** capacitor charged to **5.0 V**,

(b) A **2000 pF** capacitor charged to **10.0 V**,

(c) A **200 μF** capacitor charged to **250 V**.

2 A **50 000 μF** capacitor is charged from a **9.0 V** battery and then discharged through a light bulb in a flash of light lasting **0.2 s**. Calculate :

(a) The **charge** and **energy stored** in the capacitor before discharge,

(b) The **average power** supplied to the light bulb.

3 A **4700 μF** capacitor is charged to a p.d. of **10.0 V** and is then discharged across the terminals of a small electric motor. The motor has a thread attached to its spindle, and the thread supports a **0.1 N** weight. When the capacitor is discharged across the motor terminals, the motor raises the weight by **0.12 m**. Calculate :

(a) The **energy stored** in the capacitor initially,

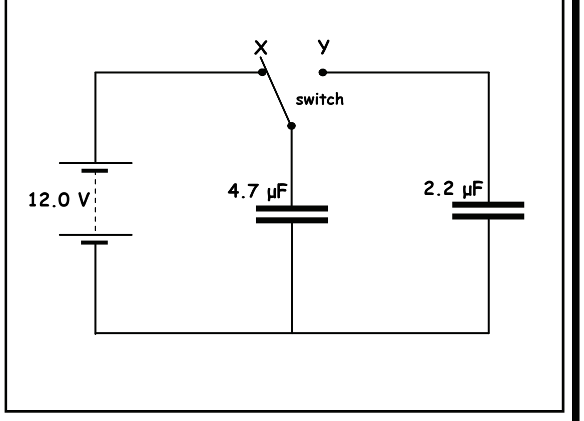
(b) The **work done** on the weight when it is raised,

(c) The **percentage efficiency** of the energy transfer.

Account for the difference between (a) and (b).

4 A **2.2 μF** capacitor is connected in **SERIES** with a **10.0 μF** capacitor and a **3.0 V** battery. Calculate the **charge** and **energy stored** in each capacitor.

5 In the circuit shown opposite a **4.7 μF** capacitor is charged from a **12.0 V** battery by connecting the switch to **X**.



The switch is then reconnected to **Y** to charge a **2.2 μF** capacitor from the first capacitor.

Calculate :

(a) The **initial charge** and **energy stored** in the 4.7 μF capacitor,

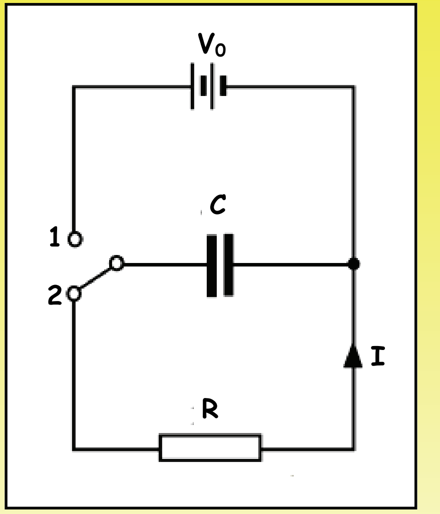
(b) The **combined capacitance** of the two capacitors,

(c) The **final p.d.** across the two capacitors,

(d) The **final energy stored** in each capacitor.

CAPACITOR DISCHARGE THROUGH A FIXED RESISTOR

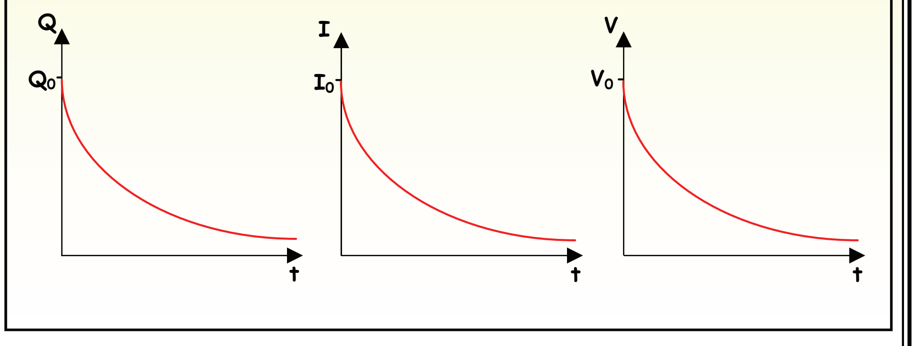
- Consider the circuit shown opposite. With the switch in position **(1)**, the capacitor is charged up by the supply.



- When the switch is moved to position **(2)** the capacitor gradually discharges through the fixed resistor **(R)**. As the discharge occurs, the following quantities decrease :

  - The **CHARGE** stored, **Q**.
  - The **CURRENT** flowing, **I**.
  - The **P.D. or VOLTAGE** across the capacitor, **V**.

Q, I and **V** all decrease **EXPONENTIALLY** with time as the capacitor discharges. This is shown in the three graphs below.



ANALYSIS OF CAPACITOR DISCHARGE GRAPHS

- When the charged capacitor is connected to the resistor (R), electrons start flowing around the circuit.

  If the **Q** stored is **high**, the **V** across the capacitor will be **high** and so the initial discharge **I** is also **high**.

  As **Q decreases** so too does V and **I**, but the **rate of decay decreases with time**.

- **MATHEMATICALLY** : $Q = CV$ and $V = IR$
  So $V \propto Q$ and $I \propto V$

Therefore, **If Q is high, V is high and so I is high.**

gradient of the Q/t graph = CURRENT (I)

At first the **gradient is steep**, so the **current is high**.
Later, the **gradient has decreased** and so the **current is less**.

area enclosed by the I/t graph = charge (Q) that flowed from the capacitor

At first **I is high**, so the area under the graph is **large**, which means that a **large amount of charge** has flowed from the capacitor.

Later, **I has decreased**, so the area under the graph is **smaller**, which means that **less charge** has flowed out of the capacitor.

CAPACITOR DISCHARGE EQUATIONS

- The **Q/t**, **I/t** and **V/t** graphs may be represented by the following discharge equations :

$$Q = Q_0 e^{-t/CR}$$

$$I = I_0 e^{-t/CR}$$

$$V = V_0 e^{-t/CR}$$

- These are all examples of the general equation :

$$x = x_0 e^{-t/CR}$$

- **Q, I, V** = Values of **charge, current and p.d.** at any time (t).
- Q_0, I_0, V_0 = Values of **charge, current and p.d.** at time, t = 0.
- **C** = **Capacitance** of the capacitor.
- **R** = **Resistance** of the resistor through which the capacitor discharges.
- **e** = **Exponential function.**

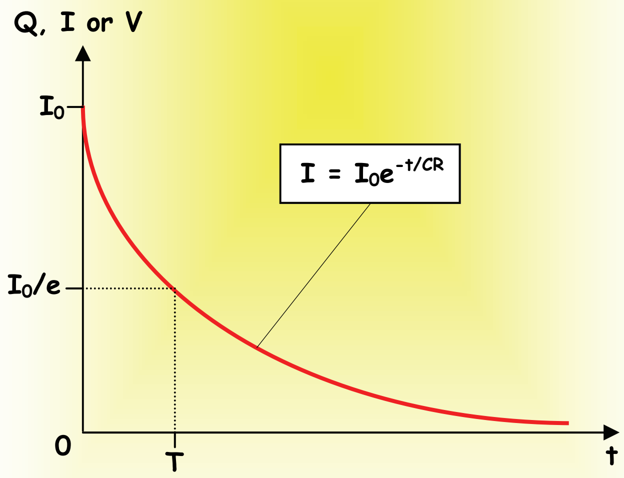
TIME CONSTANT (T)

The quantity **CR** is called the **TIME CONSTANT (T)** of the circuit.

$$T = CR$$

(s) (F) (Ω)

TIME CONSTANT (T) is the time taken for the **current (I), charge (Q) or p.d. (V)** to decrease to **1/e (≈ 37%)** of its initial value.

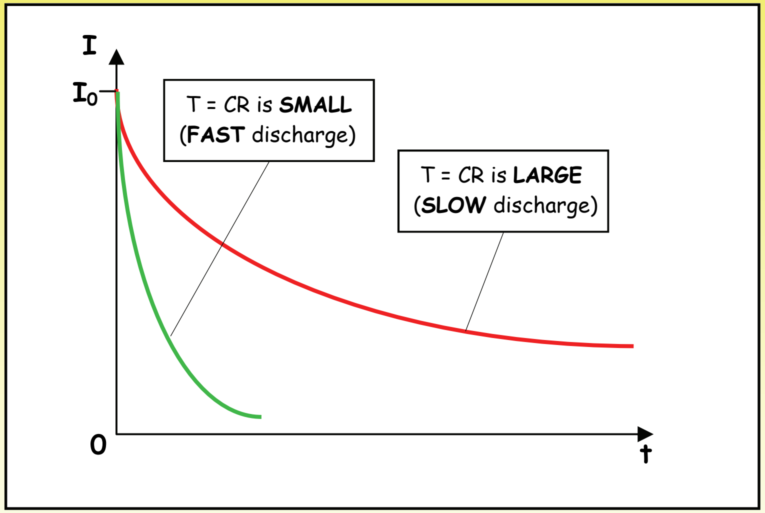


- After **t = 2T**, the value falls to $1/e^2$ of its initial value.

  After **t = 3T**, the value falls to $1/e^3$ of its initial value.

- The value of the **TIME CONSTANT (T = CR)** can be increased by **increasing** the value of **C or R or both C and R**.

- If **CR** is **large**, the discharge will be **slow** and if it is **small** the discharge will be **fast**. This is shown in the diagram below.



- Circuits in which a capacitor discharges through a resistor are often used in **electronic timers**. At the start of a time interval, the capacitor is charged up. It gradually discharges, and eventually falls below a set (Q, I,or V) value. This triggers a switching circuit to make an alarm sound, or to have some other effect.